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Method to Measure Displacement and Velocityfrom Acceleration Signals

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ABSTRACT: This paper discusses a methodology for measuring the displacement and velocity vibration of a structure from a noisy acceleration signal. Digital FIR (finite impulse response)filters were used to remove the noise present in the signal as well to eliminate numerical integration problems which produce offsets in velocity and displacement amplitudes. The choice of this type of filter aims to prevent the signal delay that may cause distortions in numerical integration process. To validate this methodology the defined procedure was applied to determine the deflection history of a beam with random excitation base. The acceleration signals considered as input data were obtained from two piezoelectric accelerometers used to measure the vibration of the base and the beam response. The correlation was defined between the stress measured by a strain gage and the stress calculated from its deflection. The result was great, enabling the application of this methodology to determine the history of displacement and velocity of the structure's vibration, through signals measured with accelerometers.

Keywords: Accelerometer signal integration, Digital filter, Displacement and velocitymeasurements.

I. INTRODUCTION

The displacement and velocitydata acquisition history is very important in vibration testing of a structure, as well their deformed condition, the study of the suspensions courses and the evaluation of working velocity for shock absorber.

The use of displacement and velocity transducers generally becomes impractical due to the need to assemble a fixed referential and also in cases are needed of high accuracy transducers who has high prices (ex. Laser). In these cases the use of acceleration sensors, such as piezoelectric accelerometers, is a viable solution for vibration data acquisition, since it is not necessary a fixed referential. However the conversion of acceleration signals in velocity and displacement is not as simple as they seem, errors in numerical integration process generate signal distortion. This paper discusses a methodology for obtaining velocity and displacement from an acceleration signal using FIR filters for correction of the numerical integration.

II. NUMERICAL INTEGRATION

There are many methods in the literature for obtaining the numerical integration of a signal, such as the rectangle rule, trapezoid, Simpson, etc. In this work was considered the trapezoidal rule for defining the numerical integration, because to be easy to implement and provides a good accuracy for high acquisition rates.

$$\begin{cases} F(t_0) = 0 \\ F(t_n) = \sum_{i=0}^{n} \left[\frac{f(t_{n-1})+f(t_n)}{2}\right].\Delta t \end{cases} \quad (1)$$

The equation (1) show the numerical integration formula was considered in this work, as can be seen, $F(t)$is the numerical integral of the function$f(t)$. For the initial condition was considered to be zero.

III. FILTER

Filters are involved in various parts of a digital signal processing system. Acting on digital signals of all kinds, such as mechanical vibration, sound, image, video, etc. [1]. Filters are invariant linear systems in time able to modify the characteristics of the signals associated in its input in such way that only a specific part of the frequency signal frequency component arrives at the filter output [2]. Those filters can be analog or digital.

3.1. Analog filters

Analog filters can be classified according to how they act in the frequency domain: Low-Pass, High-Pass, Band-Pass and Band-Stop.

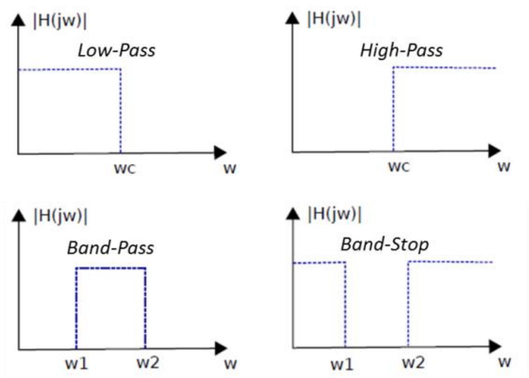

Figure1: Ideal response for the four filter types: Low-Pass, High-Pass, Band-Pass and Band-Stop.

In practice it is not possible to realize the perfect filter shown in Figure 1, the transition zones are softer with the real filters, the higher the complexity of the filter (order) closest the ideal filter will be his response [2].The Figure 2 shows four types of the low-pass filter response (Butterworth, Chebyshev Type 1, Chebyshev Type 2 and Elliptic). As can be seen, each filter has its own peculiarities, which may suit best for each type of problem.

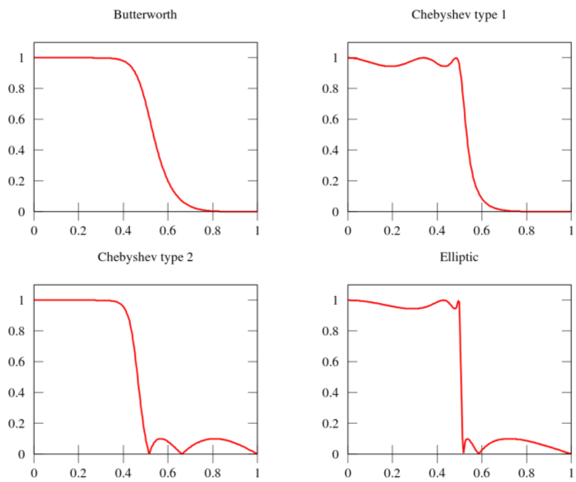

Figure2: Response four types of low-pass filters: Butterworth, Chebyshev Type 1, Type 2 Chebyshev and Elliptic.

3.2. Digital Filter

The digital filter is the implementation of a mathematical algorithm in hardware or software that operates on the signal x(n) applied to its input to generate an output filtered version y(n) [2].

Among the main advantages of digital filter stand out [2]:

- The performance is independent of the circuit components, ie, the response is not influenced by environmental changes;
- They can provide linear phase;
- The frequency response can be easily modified;
- Can be used for signals with lower frequencies.

The main disadvantage of the digital filter in comparison to analog filter is the response velocity, due to the need for conversion from analog signal to digital [2].

Digital filters are classified according to the length of the response to the impulse: FIR (finite impulse response filter) and IIR (infinite response filter to the impulse). The choice between them depends on the specific application, being necessary to evaluate their characteristics. The filter transfer function of the FIR filter and IIR are presented in equation (2) and (3), respectively.

$$H(z) = \frac{\sum_{k=0}^{N} b_k z^{-k}}{1+\sum_{k=1}^{M} a_k z^{-k}} \qquad (2)$$

$$H(z) = \sum_{k=0}^{N-1} h(k) z^{-k} \qquad (3)$$

The Filters IIR Advantages

- IIR digital filters in general require less coefficient than FIR to attend the same design specifications, therefore have a lower computational cost [2];
- Analog filters can be easily converted into equivalent digital IIR filters [2];

The Filters FIR Advantages

- They have linear phase response, so there is not phase distortion in the filtered signal [2];
- They are not performed recursively, and so are always stable, which cannot be ensured for the IIR filter [2];
- The error effects for the finite precision and the quantization are less severe for FIR filters [2].

IV. ERRORS INHERENT IN DIRECT INTEGRATION PROCESS

Initially, to assess the problem generated by the integration process "directly", without applying correction filters, it will be considered a simple sinusoidal acceleration signal with frequency of 1 Hz and amplitude 1 m/s, defined by equation (4).

$$a(t) = sen(2.\pi.t) \quad (4)$$

The velocity of the acceleration signal can be analytically obtained through integration, as shown in equation (5).

$$v(t) = -\frac{1}{2.\pi}.cos(2.\pi.t) + C1 \qquad (5)$$

Similarly, the displacement is obtained by the integral of velocity equation, which is presented by the equation (6).

$$u(t) = -\frac{1}{(2.\pi)^2}.sen(2.\pi.t) + C_1.t + C_2 \qquad (6)$$

As can be seen, there is no single analytical solution for velocity and displacement, requiring knowledge of other information to set the constant values of C_1andC_2.For this example the constants will be considered equal to zero, representing a vibration system with zero mean for velocity and displacement, which is indicated an initial condition $v(0) = -\frac{1}{2.\pi}$and$u(0) = 0$.

The Figure3 shows a comparison of velocity and displacement obtained analytically (real) with the numerically calculated by trapezoidal rule. For a signal 10s with the data acquisition rate of 512Hz.

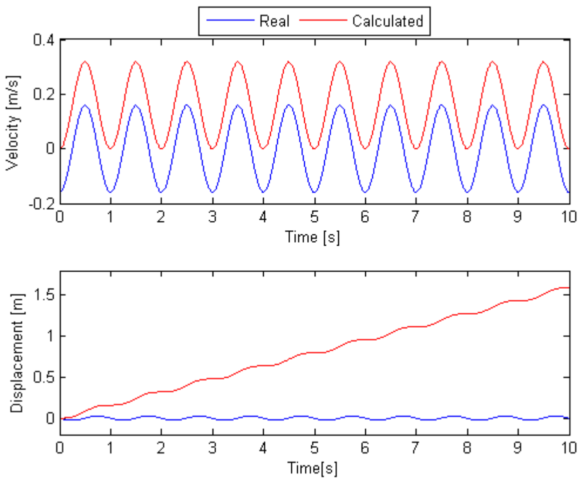

Figure3: Comparison of velocity and displacement for the analytical solution (real) and numerically calculated without application filter.

As can be seen, the numerical solution shows variations of the analytical solution (real). In graphic of the velocity is observed a constant offset between the calculated signal and the real, since for the displacement the offset increases linearly with time. This problem occurs because the initial condition of velocity and displacement has been considered equal to zero in the numerical integration, which is different from the real conditions.

These errors can be corrected by applying the correct initial condition for the numerical simulation, but how these values are rarely known in real vibration conditions, this correction is impracticable.

Another way to fix this problem is the application of a high pass filter to remove this offset, but this method can only be used on signs that have zero mean vibration. Because most vibration problems are classified in this condition, in this paper this hypothesis will be considered.

V. DIGITAL FILTER DESIGN FOR CORRECTION OF THE NUMERICAL INTEGRATION FOR SIGNALS

The procedure considered for converting the acceleration signal into velocity and displacement with the digital filters applied for removing the constant component in each integration can be observed in the flowchart shown in Figure 4.

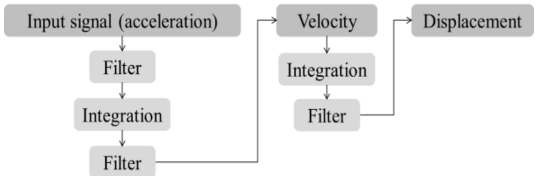

Figure4: Flowchart used to calculate the velocity and displacement from anacceleration signalwithoutnoise, using digital filters for removing constant components.

The filter considered in this paper is similar to that proposed by JoséRibeiro in his doctoral thesis, an FIR high-pass filter with equivalent curve to the Chebyshev type I filter curve, in order to obtain a shorter transition band with cutoff maximum frequency of 1Hz [3]. TheFigure5 shows the frequency response and the filter impulse response for the correction of numerical integration, which was defined according to the specifications listed below.

- Passband corner Frequency: 1Hz;
- Stopband corner Frequency: 0,25Hz;
- Max. attenuation on passband: 0,01dB;
- Min. attenuation on stopband: 20dB

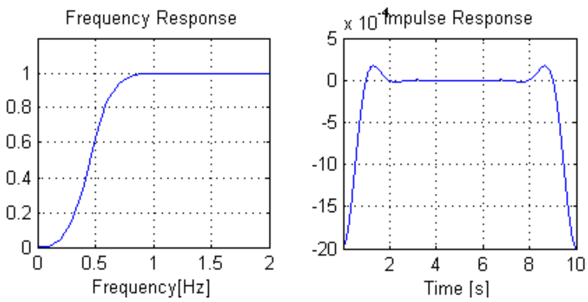

Figure5: Frequency response and the impulse response for the High-Pass FIR filter proposed for the correction of errors inherent to numerical integration process.

5.1. Signal distortion due to loss in phase

The choice of filter is fundamental to thesuccess of this conversion process, the loss in phase generated by some filters results in the distortion of the velocity and displacement signals. To illustrate this problem, the velocity and displacement signals corrected by proposed FIR filter (without distortion in phase) and by an IIR Butterworth filter (with distortion in phase) with equivalent specification were compared.

Firstly for the comparison of the two filters (FIR & IIR) was considered a sine acceleration signal that contains only one frequency, described by equation (4).

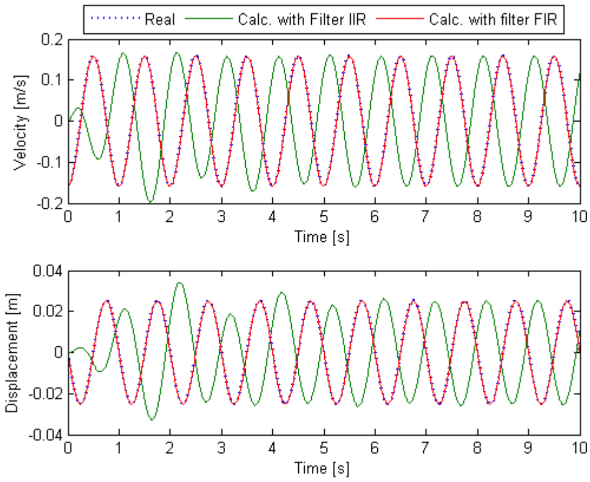

Figure6: Comparison of velocity and displacement corrected by FIR and IIR filter from a sine single frequency acceleration signal (1Hz).

As can be seen in the Figure 6, the FIR filter showed excellent results for velocity and displacement calculation, overlapping the analytic solution (real). The signals calculated by IIR filter showed distortion in the firsts seconds and a loss of phase when compared with analytic solution.

To evaluate the performance of the two filters, for multiple frequencies signals, was adopted a sine acceleration signal composed by three frequencies: 1Hz, 3Hz and 9Hz,with amplitude of 1m/s², 2m/s² and 3m/s² respectively, described by equation (7).

$$a(t) = sen(2.\pi.t) + 2.sen(2.\pi.3.t) + 3.sen(2.\pi.9.t) \quad (7)$$

The Figure 7 shows the comparative between the two filters used for correction of velocity and displacement signals. As already expected, the signals achieved by FIR filter showed excellent result, without distortion of the signal. Whereas the signals obtained by IIR filter showed distortion in the whole signal.

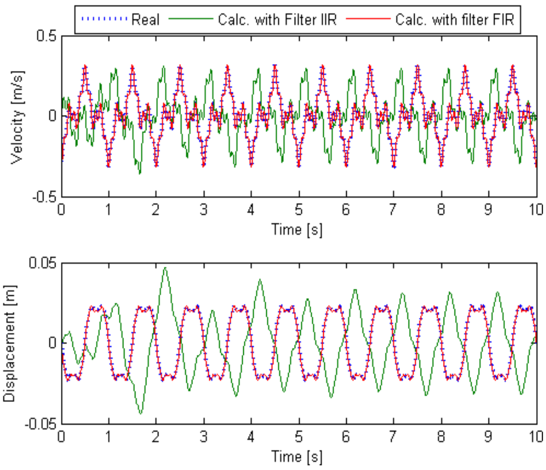

Figure7: Comparison of velocity and displacement corrected by FIR and IIR filter from an acceleration sine signal composed by three frequencies (1, 3 and 9Hz).

5.2. Time Aliasing Problem

According to observed in the figures 6 and 7 the velocity and displacement signals corrected by FIR filter perfectly coincide with the expected. This occurs due to the frequency of both signals were specified in FIR filter. In case there is a signal frequency that is not specified the time aliasing will occur.

The FIR filter step depends only of the signal length, being defined by the inverse of the signal time. In case of evaluated signals in this paper, that are defined in 10s, the FIR filter step is 0,1Hz. Therefore to show the error of time aliasing was considered an acceleration signal equivalent to equation (7), however with frequencies of 1.05Hz,3.05Hz and 9.05Hz, not sampled in FIR filter.

To reduce the error from time aliasing was applied a truncation at the initial and final stretch of the signal, before the filter process, using a curve equivalent to 5th order Butterworth filter [3], the flowchart considered to the velocity and displacement computations using this truncation is presented in figure 8.

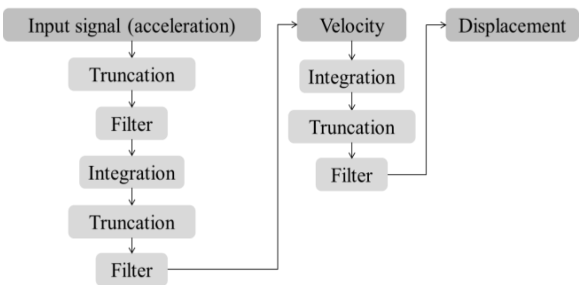

Figure8: Considered flowchart to calculate velocity and displacement from acceleration signal without noise, using digital filters to remove constants

components and the truncation to reduce errors due to time aliasing.

The figure 9 presents the velocity and displacement signals achieved by this truncation. As can be seen, the computed velocity and displacement in both condition shows some errors in initial and final stretch of the signal, however in the signals where truncation was applied the stretch with errors were shorter.

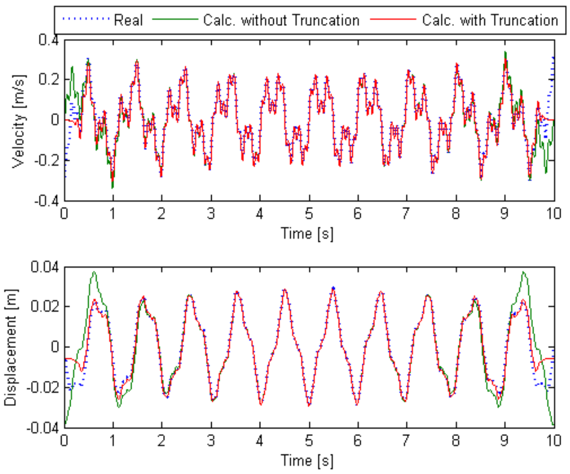

Figure9: Velocity and displacement corrected by FIR filter with and without truncation for a signal composed by three frequencies not sampled in FIR filter.

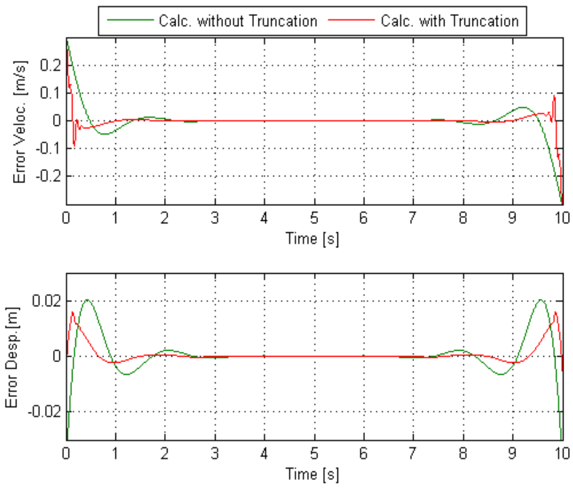

Figure10: Comparison between the error due to time aliasing in velocity and displacement with and without truncation.

The length of initial and final signal's stretch affected by time aliasing can be predicted before filter process, this is direct related with filter's impulse response showed in Figure5.The impulse response stabilize after 2s and the same error is obtained in velocity and displacement calculation without truncation observed inFigure10.

VI. VELOCITY AND DISPLACEMENT CALCULATION IN ACCELERATION NOISY SIGNALS

The most acceleration signals measured by piezoelectric accelerometers show acquisition noise, so filters application is capital to remove noise and achieve desired signal. However some filters can generate lack in phase of the filtered signal resulting distortion for calculated velocity and displacement signals. Therefore in this topic is presented a methodology for determination of velocity and displacement removing non desired frequencies.

For the evaluation of this methodology is considered an analytic acceleration signal more elaborated, with the combination of sine signals with different phases and a time varying amplitude, the aim is to represent a random signal. The equation (8) show acceleration signal considered.

$$\begin{cases} a(t) = \sum_{i=1}^{5} a_i \\ a_i(t) = A_i.\text{sen}(2.\pi.f_{A_i}.t).\text{sen}(2.\pi.f_i.t + \varphi_i) \end{cases} \quad (8)$$

$$\begin{vmatrix} A_1 = 2; A_2 = 3; A_3 = 5; A_4 = 15; A_5 = 5; \\ f_{A_1} = 0.1; f_{A_2} = 0.2; f_{A_3} = 1.5; f_{A_4} = 0.5; f_{A_5} = 2; \\ f_1 = 2.05; f_2 = 4.05; f_3 = 7.05; f_4 = 10.05; f_5 = 12.05; \\ \varphi_1 = 2; \varphi_2 = 3; \varphi_3 = 7; \varphi_4 = 4; \varphi_5 = 5; \end{vmatrix}$$

From acceleration signal defined in equation (8) were added two sine signals that represent the noise to be removed: on composed by 20Hz frequency and 2m/s² of amplitude and the other with 30Hz frequency and 5m/s².

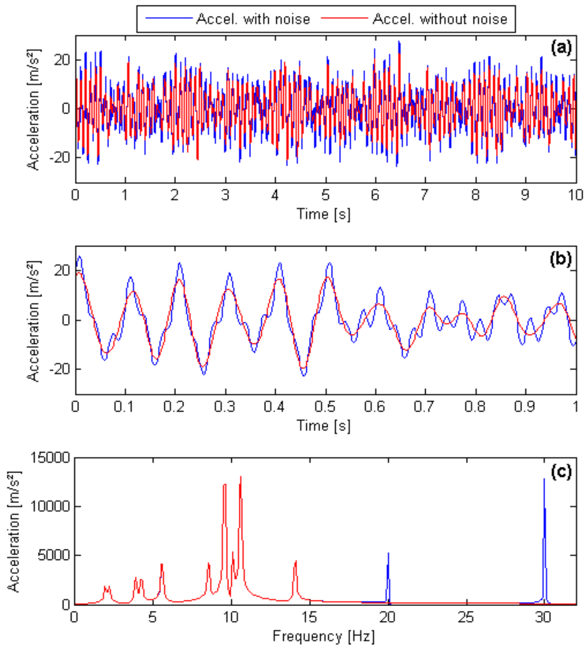

Figure11: Comparative between the signals with and without noise: (a) signals in the time domain; (b)

zoom (0-1s) signals in the time domain; (c) signals in the frequency domain.

The Figure11 shows the comparative between the signals with and without noise, in the charts (a) and (b) the signals are presented in time domain and in thechart (c) in frequency domain. Looking at frequency domain is possible to see the two peaks in blue that correspond to the inserted noise.

Before apply the defined procedure to calculate velocity and displacement, presented in Figure8, is necessary to remove the noise inside the acceleration signal. The procedure considered for the velocity and displacement calculation from acceleration noisy signals is presented in Figure12.

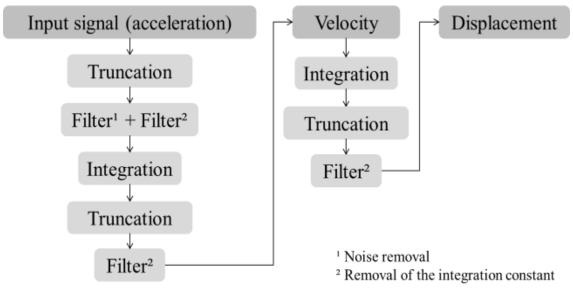

Figure12: Flowchart considered for the velocity and displacement calculation from an acceleration noisy signal, using digital filters to remove constants components and the truncation to reduce errors due to time aliasing.

To remove the noise at 20Hz and 30Hz observed in Figure11, was designed a FIR lowpass filter, since that all frequencies of interest are lower than noise frequencies. The considered parameters for filter design can be seen in sequence.

- Passband corner Frequency: 15Hz;
- Stopband corner Frequency: 20Hz;
- Max. attenuation on passband: 0,01dB;
- Min. attenuation on stopband: 20dB

TheFigure13present the frequency and impulse response for the proposed filter to remove noise at 20Hz and 30 Hz in acceleration signal.

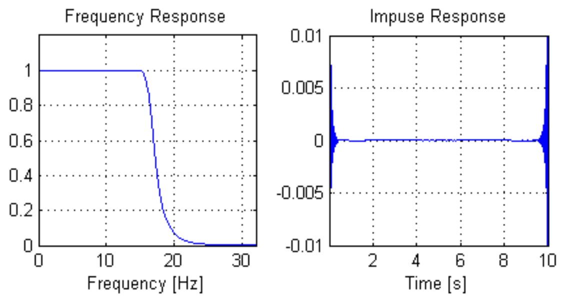

Figure13: Frequency and Impulse response of the lowpass FIR filter to remove the noise on signal.

The FIR filter designed tom remove the noise have performed excellent result, as can be seen onFigure14andFigure15, the numerically velocity and displacement signals had good relationship with the expected, only the initial and final stretch shows distortion due to time aliasing.

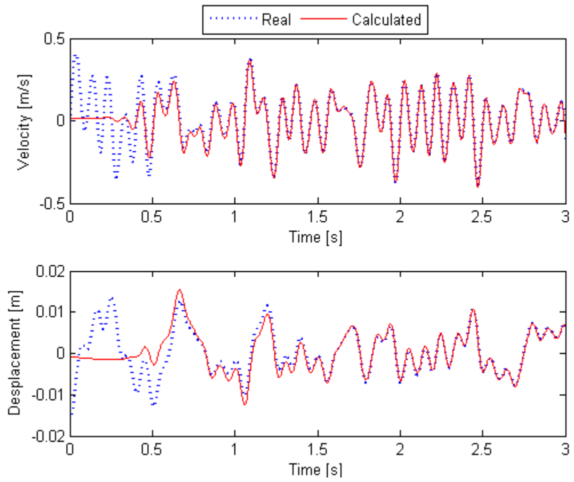

Figure14: Comparative between analytic velocity and displacement with the calculated from acceleration signal with noise at 20Hz and 30Hz (first three seconds).

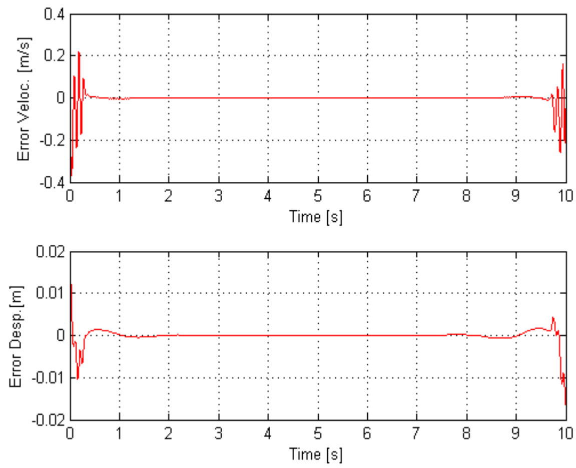

Figure15: Error of velocity and displacement signals numerically achieved from acceleration signal with noise at 20Hz and 30Hz.

VII. MEASUREMENT OF DISPLACEMENT WITH PIEZOELECTRIC ACCELEROMETERS

To validate this methodology for a real discrete acceleration signal, this methodology was applied to determine the deflection history of a beam with random base excitation applied by an electrodynamic shaker. The data acquisition vibration of the beam was realized with two piezoelectric accelerometers, one fixed to the base of the beam (A1) and the other on the concentrated mass in the free edge at end of the beam (A2). In the test was also performed data acquisition of the stress history by a uniaxial strain gage fixed to the beam base. In Figure16 shows the test configuration.

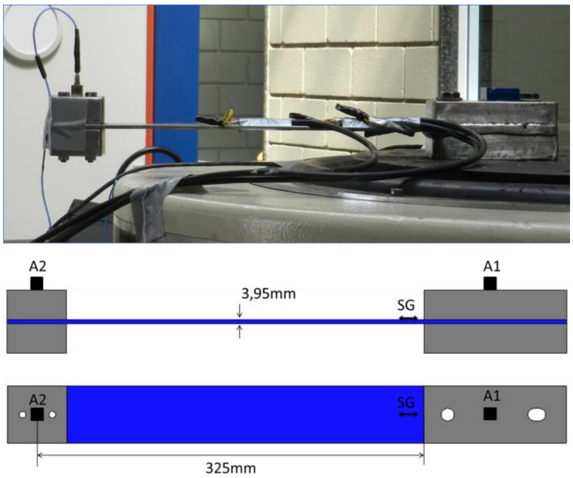

Figure16: Configuration the random vibration test for a beam.

The acceleration signals measured in the test and their respective FFTs are shown in Figure17. By the FFT of base acceleration (A1) is observed that the signal show excitation at all frequencies and the maximum amplitude is close to 10Hz frequency. Through the FFT of the mass acceleration (A2) is possible to see that response occurs mainly at the 10,8Hz frequency, which is the beam's natural frequency. The other small peak at 20Hz is due to an acquisition noise present in this signal, which must be removed.

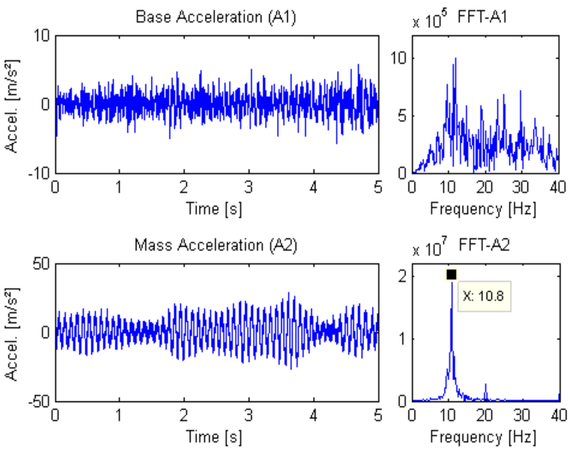

Figure17: Base acceleration signal (A1) and mass acceleration signal (A2) measured at the random vibration test.

To calculate the displacement signal of the base (U1) and mass (U2) was consider the procedure illustrated in Figure12. The low-pass filter was usedto remove acquisition noise and the high-pass filter to removal the errors generated in the numerical integration at low frequencies. The parameters of the two filters used are presented below:

High-Pass Filter:

- Passband corner Frequency: 5Hz;
- Stopband corner Frequency: 3Hz;
- Max. attenuation on passband: 0,01dB;
- Min. attenuation on stopband: 20dB

Low-Pass Filter:

- Passband corner Frequency: 15Hz;
- Stopband corner Frequency: 20Hz;
- Max. attenuation on passband: 0,01dB;
- Min. attenuation on stopband: 20dB

Figure18shows that the displacement signals of the base and the mass obtained from the acceleration signals measured in the test. From the difference between the two signals was defined beam historical deflection, which is can be seen from Figure19.

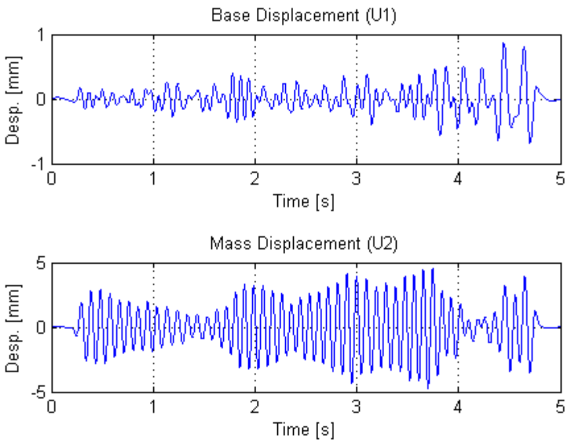

Figure18: The Base Displacement (U1) and mass displacement (U2) calculated from the accelerations signals obtained in random vibration test.

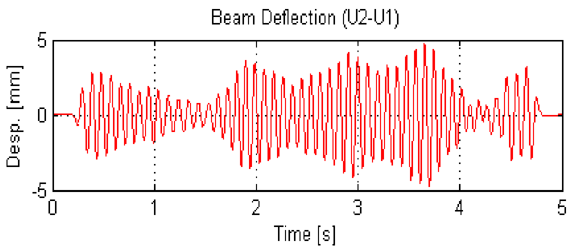

Figure19: Beam deflection history at random test.

For ensure that the deflection shown in Figure19 is correct. It was calculated a stress history of the beam base using the equation (9), which correlates the deflection (f)shown in Figure19 with the stress in the base of the beam, thus allowing its comparison with the stress history measured with strain gage.

$$\sigma(t) = \left(\frac{-3.h.E}{2.L^2}\right).f(t) = -11{,}78.f(t) \qquad (9)$$

$$\left| \begin{array}{l} E: YoungModulus = 210GPa \\ h: BeamThickness = 3{,}95mm \\ L = BeamLength = 325mm \end{array} \right.$$

The Figure20 shows the comparison between the measured stress history and the calculated one. In the upper graphic one can see the good agreement of the results. The difference between them is shown in the graphic below. The largest variation can be observed in the initial stretch (0-0.3 s) and in the final stretch (4.7-5.0 s), due to the time aliasing. Disregarding these stretches, the mean relative error between the peaks was approximately 5.3%, which is very small.

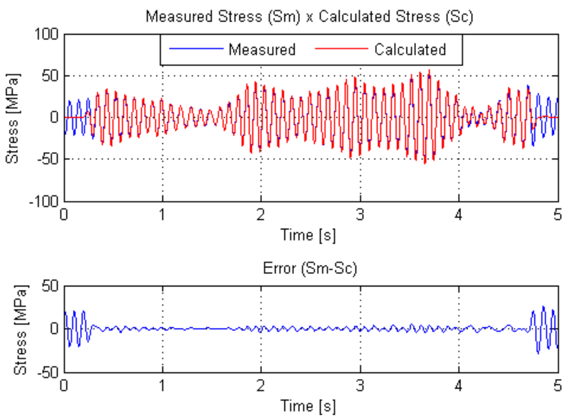

Figure20: Comparison between the measured stress signal to the stress signal measured across the beam deflection.

VIII. CONCLUSION

The procedure used to calculate velocity and displacement from a discreet acceleration signal, with and without noise acquisition, presented great results. Although the parameters used to define the filters should be carefully defined in a way to eliminate the noises and the offsets without removing frequency ranges relevant to the evaluated signal.

The issue of time aliasing is inevitable for the type of filter used, which generates a signal distortion in the beginning and in the end of the signal. The truncation of the signal can reduce the length of the affected part, although it won't completely eliminate the error. Therefore, to solve this problem completely, it is recommended that the measured signal owns an initial and final part that can be removed after the velocity and displacement calculation. The signal length to be disregardedmay be predicted through the filter impulse response.

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